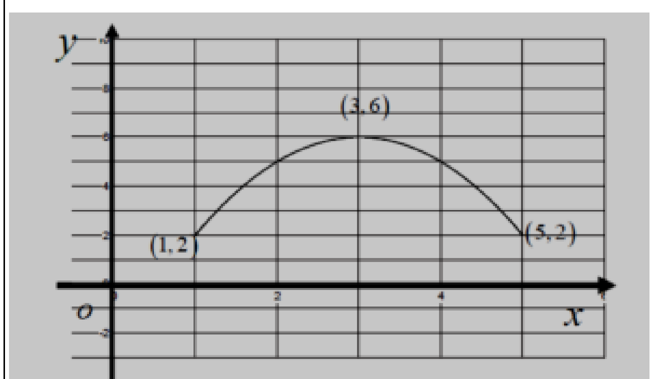
Parametric Equations- MS

June 2018 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1-2\sin^2 t \Rightarrow \frac{y-4}{2} = 1-2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y-4 = 2-4\times\frac{(x-3)^2}{4} \Rightarrow y = 6-(x-3)^2$ *	A1*	1.1b
		(2)	
(b)	 $\cap$ shaped parabola	M1	1.1b
	Fully correct with 'ends' at (1,2) & (5,2)	A1	1.1b
	Suitable reason : Eg states as $x = 3+2\sin t$, $1 \leqslant x \leqslant 5$	B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a
	Finds where $x+y=k$ meets $y = 6-(x-3)^2$ $\Rightarrow k-x = 6-(x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $\quad x^2-7x+(k+3)=0$ Or y $\quad y^2+(7-2k)y+(k^2-6k+3)=0$	A1	1.1b

	Uses $b^2-4ac=0 \Rightarrow\ 49-4\times1\times(k+3)=0 \Rightarrow k=\left(\frac{37}{4}\right)$ or $(7-2k)^2-4\times1\times(k^2-6k+3)=0 \Rightarrow k=\left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k=\left\{k: 7\leqslant k<\frac{37}{4}\right\}$	A1	2.5
		(5)	
			(10 marks)

(a)

M1: Uses $\cos 2t=1-2\sin^2 t$ in an attempt to eliminate t

A1*: Proceeds to $y=6-(x-3)^2$ without any errors

Allow a proof where they start with $y=6-(x-3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t=1-2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar .

(b)

M1: For sketching a $\cap$ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a $\cap$ parabola with a maximum in quadrant one and with end coordinates of $(1,2)$ and $(5,2)$

B1: Any suitable explanation as to why C does not include all points of $y=6-(x-3)^2,\quad x\in\mathbb{R}$

This should include a reference to **the limits on sin or cos** with a **link to a restriction on *x* or *y***.

For example

'As $-1\leq\sin t\leq1$ then $1\leqslant x\leqslant5$' Condone in words 'x lies between 1 and 5' and strict inequalities

'As $\sin t\leq1$ then $x\leqslant5$' Condone in words 'x is less than 5'

'As $-1\leq\cos(2t)\leq1$ then $2\leqslant y\leqslant6$ ' Condone in words 'y lies between 2 and 6'

Withhold if the statement is incorrect Eg "because the domain is $2\leqslant x\leqslant5$"

Do not allow a statement on the top limit of y as this is the same for both curves

(c)

B1: Deduces either

- the correct that the lower value of $k=7$ This can be found by substituting into $(5,2)$

  $x+y=k\Rightarrow k=7$ or substituting $x=5$ into $x^2-7x+(k+3)=0\Rightarrow 25-35+k+3=0$
  $\Rightarrow k=7$

- or deduces that $k<\frac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for k.

Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method.

Eg. Sets $k-x=6-(x-3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2-7x+(k+3)=0$ The $=0$ may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2=4ac$ oe or $b^2...4ac$ where ... is any inequality leading to a critical value for k. Eg. one root $\Rightarrow 49-4\times1\times(k+3)=0 \Rightarrow k=\frac{37}{4}$

A1: Range of values for $k=\left\{k:7\leqslant k<\frac{37}{4}\right\}$ Accept $k\in\left[7,\frac{37}{4}\right)$ or exact equivalent

ALT	As above	B1	2.2a
	Finds where $x+y=k$ meets $y=6-(x-3)^2$ once by using an appropriate method. Eg. Sets gradient of $y=6-(x-3)^2$ equal to -1	M1	3.1a
	$-2x+6=-1 \Rightarrow x=3.5$	A1	1.1b
	Finds point of intersection and uses this to find upper value of k. $y=6-(3.5-3)^2=5.75$ Hence using $k=3.5+5.75=9.25$	M1	2.1
	Range of values for $k=\{k:7\leqslant k<9.25\}$	A1	2.5

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2.

Question Number	Scheme	Notes	Marks
1.	$x=3t-4,\ y=5-\frac{6}{t},\quad t>0$		
(a)	$\frac{dx}{dt}=3,\ \frac{dy}{dt}=6t^{-2}$		
	$\frac{dy}{dx}=\frac{6t^{-2}}{3}\ \left\{=\frac{6}{3t^2}=2t^{-2}=\frac{2}{t^2}\right\}$	their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ to give $\frac{dy}{dx}$ in terms of t **or** their $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$ to give $\frac{dy}{dx}$ in terms of t	M1
		$\frac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t. **See note.**	A1 isw
	Award **Special Case 1st M1** if **both** $\frac{dx}{dt}$ **and** $\frac{dy}{dt}$ are stated **correctly** and **explicitly**.		**[2]**
	Note: You can recover the work for part (a) in part (b).		
(a) **Way 2**	$y=5-\frac{18}{x+4} \Rightarrow \frac{dy}{dx}=\frac{18}{(x+4)^2}=\frac{18}{(3t)^2}$	Writes $\frac{dy}{dx}$ in the form $\frac{\pm\lambda}{(x+4)^2}$, ***and*** writes $\frac{dy}{dx}$ as a function of t.	M1
		Correct un-simplified or simplified answer, in terms of t. **See note.**	A1 isw
			[2]

(b)	$\left\{t=\frac{1}{2}\Rightarrow\right\}P\left(-\frac{5}{2},-7\right)$	$x=-\frac{5}{2},\ y=-7$ **or** $P\left(-\frac{5}{2},-7\right)$ seen or implied.	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y-\text{"}-7\text{"}=\text{"}8\text{"}\left(x-\text{"}-\frac{5}{2}\text{"}\right)$ • $\text{"}-7\text{"}=(\text{"}8\text{"})(\text{"}-\frac{5}{2}\text{"})+c$ So, $y=(\text{their } m_T)x+\text{"}c\text{"}$	**Some** attempt to substitute $t=0.5$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which contains t in order to find m_T **and either** applies $y-(\text{their } y_P)=(\text{their } m_T)(x-\text{their } x_P)$ **or** finds c from $(\text{their } y_P)=(\text{their } m_T)(\text{their } x_P)+c$ and uses their numerical c in $y=(\text{their } m_T)x+c$	M1
	T: $y=8x+13$	$y=8x+13$ or $y=13+8x$	A1 **cso**
	Note: their x_P, their y_P and their m_T must be numerical values in order to award M1		**[3]**
(c) **Way 1**	$\left\{t=\frac{x+4}{3}\Rightarrow\right\}y=5-\frac{6}{\left(\frac{x+4}{3}\right)}$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	Þ $y=5-\frac{18}{x+4}$ Þ $y=\frac{5(x+4)-18}{x+4}$		
	So, $y=\frac{5x+2}{x+4}$, $\{x>-4\}$	$y=\frac{5x+2}{x+4}$ (or implied equation)	A1 **cso**
			[3]
(c) **Way 2**	$\left\{t=\frac{6}{5-y}\Rightarrow\right\}x=\frac{18}{5-y}-4$	An attempt to eliminate t. **See notes.**	M1
		Achieves a correct equation in x and y only	A1 o.e.
	Þ $(x+4)(5-y)=18$ Þ $5x-xy+20-4y=18$		
	$\{$Þ $5x+2=y(x+4)\}$ So, $y=\frac{5x+2}{x+4}$, $\{x>-4\}$	$y=\frac{5x+2}{x+4}$ (or implied equation)	A1 **cso**
			[3]
	Note: Some or all of the work for part (c) can be recovered in part (a) or part (b)		**8**

Question Number	Scheme	Notes	Marks
1. (c) **Way 3**	$y=\frac{3at-4a+b}{3t-4+4}=\frac{3at}{3t}-\frac{4a-b}{3t}=a-\frac{4a-b}{3t}$ Þ $a=5$	A full method leading to the value of a being found	M1
		$y=a-\frac{4a-b}{3t}$ and $a=5$	A1
	$\frac{4a-b}{3}=6\Rightarrow b=4(5)-6(3)=2$	**Both** $a=5$ **and** $b=2$	A1
			[3]

		Question 1 Notes
1. (a)	**Note**	Condone $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(\frac{6}{t^2}\right)}{3}$ for A1
	Note	You can ignore subsequent working following on from a correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.
(b)	**Note**	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}}$ or $\frac{1}{\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}}$ or $-\left(\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}\right)$) is M0.
	Note	**Final A1:** A correct solution is required from a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	Note	**Final A1:** You can ignore subsequent working following on from a correct solution.
(c)	**Note**	**1st M1:** A full attempt to eliminate t is defined as either • rearranging one of the parametric equations to make t the subject and substituting for t in the other parametric equation (only the RHS of the equation required for M mark) • rearranging both parametric equations to make t the subject and putting the results equal to each other.
	Note	Award M1A1 for $\frac{6}{5-y}=\frac{x+4}{3}$ or equivalent.

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3.

Question Number	Scheme	Notes	Marks
5.	$x = 4\tan t,\quad y = 5\sqrt{3}\sin 2t,\qquad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 1**	$\frac{dx}{dt} = 4\sec^2 t,\ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t}\quad \left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$	**Either both** x and y are differentiated correctly with respect to t **or** their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **or** applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
		Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	**dependent on the previous M mark** ***Some evidence*** of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 cso
			[4]
(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right),\ y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ **or** $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ **or** $x = 4$ **or** $y = 5\sqrt{3}$ **or** $y =$ awrt 8.7	M1
	Coordinates are $(4, 5\sqrt{3})$	$(4, 5\sqrt{3})$ **or** $x = 4,\ y = 5\sqrt{3}$	A1
			[2]
			6

		Question 5 Notes
5. (a)	**1st A1**	Correct $\frac{dy}{dx}$. E.g. $\frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ or $\frac{5}{2}\sqrt{3}\cos 2t\cos^2 t$ or $\frac{5}{2}\sqrt{3}\cos^2 t(\cos^2 t - \sin^2 t)$ or any equivalent form.
	Note	Give the final A0 for a final answer of $-\frac{10}{32}\sqrt{3}$ **without reference to** $-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$
	Note	Give the final A0 for more than one value stated for $\frac{dy}{dx}$
(b)	**Note**	Also allow M1 for either $x = 4\tan(45)$ or $y = 5\sqrt{3}\sin(2(45))$
	Note	M1 can be gained by ignoring previous working in part (a) and/or part (b)
	Note	Give A0 for stating more than one set of coordinates for Q.
	Note	Writing $x = 4,\ y = 5\sqrt{3}$ followed by $(5\sqrt{3}, 4)$ is A0.

Question Number	Scheme	Notes	Marks
5.	$x = 4\tan t, \quad y = 5\sqrt{3}\sin 2t, \quad 0 \leqslant t < \frac{\pi}{2}$		
(a) **Way 2**	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{(x^2+16)}}, \ \cos t = \frac{4}{\sqrt{(x^2+16)}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{\begin{array}{ll} u = 40\sqrt{3}x & v = x^2+16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{array}\right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \ \left\{= \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2}\right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]
(a) **Way 3**	$y = 5\sqrt{3}\sin\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)$		
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{2}{1+\left(\frac{x}{4}\right)^2}\right)\left(\frac{1}{4}\right)$	$\frac{dy}{dx} = \pm A\cos\left(2\tan^{-1}\left(\frac{x}{4}\right)\right)\left(\frac{1}{1+x^2}\right)$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified.	A1
	$\frac{dy}{dx} = 5\sqrt{3}\cos\left(2\tan^{-1}\left(\sqrt{3}\right)\right)\left(\frac{2}{1+3}\right)\left(\frac{1}{4}\right) \ \left\{= 5\sqrt{3}\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right\}$	**dependent on the previous M mark** ***Some evidence*** of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ **from a correct solution only**	A1 **cso**
			[4]

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4.

Question Number	Scheme		Marks
5.	**Note: You can mark parts (a) and (b) together.**		
(a)	$x = 4t + 3,\ \ y = 4t + 8 + \dfrac{5}{2t}$		
	$\dfrac{dx}{dt} = 4,\ \ \dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	Both $\dfrac{dx}{dt} = 4$ or $\dfrac{dt}{dx} = \dfrac{1}{4}$ and $\dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$	B1
	So, $\dfrac{dy}{dx} = \dfrac{4 - \frac{5}{2}t^{-2}}{4}\ \left\{= 1 - \dfrac{5}{8}t^{-2} = 1 - \dfrac{5}{8t^2}\right\}$	Candidate's $\dfrac{dy}{dt}$ divided by a candidate's $\dfrac{dx}{dt}$	M1 o.e.
	$\{\text{When } t = 2,\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
	Way 2: Cartesian Method		
	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$	$\dfrac{dy}{dx} = 1 - \dfrac{10}{(x-3)^2}$, simplified or un-simplifed.	B1
		$\dfrac{dy}{dx} = \pm\lambda \pm \dfrac{\mu}{(x-3)^2},\ \ \lambda \neq 0, \mu \neq 0$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
	Way 3: Cartesian Method		
	$\dfrac{dy}{dx} = \dfrac{(2x+2)(x-3) - (x^2+2x-5)}{(x-3)^2}$	Correct expression for $\dfrac{dy}{dx}$, simplified or un-simplified.	B1
	$\left\{= \dfrac{x^2 - 6x - 1}{(x-3)^2}\right\}$	$\dfrac{dy}{dx} = \dfrac{f'(x)(x-3) - 1f(x)}{(x-3)^2}$, where $f(x) =$ their "$x^2 + ax + b$", $g(x) = x - 3$	M1
	$\{\text{When } t = 2, x = 11\}\ \dfrac{dy}{dx} = \dfrac{27}{32}$	$\dfrac{27}{32}$ or 0.84375 **cao**	A1
			[3]
(b)	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\frac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3}$		
	$y = \dfrac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ ***or*** $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ ***or*** $y = \dfrac{(x+5)(x-3) + 10}{x-3}$ ***or*** $y = \dfrac{(x+5)(x-3)}{x-3} + \dfrac{10}{x-3}$	See notes	dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x-3},\ \ \{a = 2 \text{ and } b = -5\}$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x-3}$ or $a = 2$ and $b = -5$	A1 cso
			[3] **6**

Question Number	Scheme		Marks
5. (b)	**Alternative Method 1 of Equating Coefficients**		
	$y = \dfrac{x^2 + ax + b}{x - 3} \Rightarrow y(x-3) = x^2 + ax + b$		
	$y(x-3) = (4t+3)^2 + 2(4t+3) - 5 = 16t^2 + 32t + 10$		
	$x^2 + ax + b = (4t+3)^2 + a(4t+3) + b$		
	$(4t+3)^2 + a(4t+3) + b = 16t^2 + 32t + 10$	Correct method of obtaining an equation in only t, a and b	M1
	t: $24 + 4a = 32 \Rightarrow a = 2$	Equates their coefficients in t and finds both $a = \ldots$ and $b = \ldots$	dM1
	constant: $9 + 3a + b = 10 \Rightarrow b = -5$	$a = 2$ and $b = -5$	A1
			[3]
5. (b)	**Alternative Method 2 of Equating Coefficients**		
	$\left\{t = \dfrac{x-3}{4} \Rightarrow\right\} y = 4\left(\dfrac{x-3}{4}\right) + 8 + \dfrac{5}{2\left(\dfrac{x-3}{4}\right)}$	Eliminates t to achieve an equation in only x and y	M1
	$y = x - 3 + 8 + \dfrac{10}{x-3} \Rightarrow y = x + 5 + \dfrac{10}{(x-3)}$		
	$\underline{\underline{y(x-3)}} = (x+5)(x-3) + 10 \Rightarrow x^2 + ax + b = \underline{\underline{(x+5)(x-3) + 10}}$		dM1
	$\Rightarrow y = \dfrac{x^2 + 2x - 5}{x - 3}$ **or** equating coefficients to give $a = 2$ and $b = -5$	Correct algebra leading to $y = \dfrac{x^2 + 2x - 5}{x - 3}$ or $a = 2$ and $b = -5$	A1 **cso**
			[3]

		Question 5 Notes
5. (a)	**B1**	$\dfrac{dx}{dt} = 4$ and $\dfrac{dy}{dt} = 4 - \dfrac{5}{2}t^{-2}$ or $\dfrac{dy}{dt} = \dfrac{8t^2 - 5}{2t^2}$ or $\dfrac{dy}{dt} = 4 - 5(2t)^{-2}(2)$, etc.
	Note	$\dfrac{dy}{dt}$ can be simplified or un-simplified.
	Note	**You can imply the B1 mark by later working.**
	M1	Candidate's $\dfrac{dy}{dt}$ divided by a candidate's $\dfrac{dx}{dt}$ **or** $\dfrac{dy}{dt}$ multiplied by a candidate's $\dfrac{dt}{dx}$
	Note	M1 can be also be obtained by substituting $t = 2$ into both their $\dfrac{dy}{dt}$ and their $\dfrac{dx}{dt}$ and then dividing their values the correct way round.
	A1	$\dfrac{27}{32}$ or 0.84375 **cao**

(b)	M1	Eliminates t to achieve an equation in only x and y.
	dM1	**dependent on the first method mark being awarded.** **Either: (ignoring sign slips or constant slips, noting that k can be 1)** • Combining all three parts of their $\underline{x-3} + \overset{=}{8} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ to form a single fraction with a common denominator of $\pm k(x-3)$. Accept three separate fractions with the same denominator. • Combining both parts of their $\underline{x+5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$, (where $\underline{x+5}$ is their $4\left(\frac{x-3}{4}\right)+8$), to form a single fraction with a common denominator of $\pm k(x-3)$. Accept two separate fractions with the same denominator. • Multiplies both sides of their $y = \underline{x-3} + \overset{=}{8} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ or their $y = \underline{x+5} + \underline{\underline{\left(\frac{10}{x-3}\right)}}$ by $\pm k(x-3)$. Note that all terms in their equation must be multiplied by $\pm k(x-3)$.
	Note	Condone "invisible" brackets for dM1.
	A1	Correct algebra with no incorrect working leading to $y = \frac{x^2+2x-5}{x-3}$ or $a = 2$ and $b = -5$
	Note	**Some examples for the award of dM1 in (b):** **dM0** for $y = x-3+8+\frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3)+8+10}{x-3}$. Should be $...+8(x-3)+...$ **dM0** for $y = x-3+\frac{10}{x-3} \rightarrow y = \frac{(x-3)(x-3)+10}{x-3}$. The "8" part has been omitted. **dM0** for $y = x+5+\frac{10}{x-3} \rightarrow y = \frac{x(x-3)+5+10}{x-3}$. Should be $...+5(x-3)+...$ **dM0** for $y = x+5+\frac{10}{x-3} \rightarrow y(x-3) = x(x-3)+5(x-3)+10(x-3)$. Should be just 10.
	Note	$y = x+5+\frac{10}{x-3} \rightarrow y = \frac{x^2+2x-5}{x-3}$ with no intermediate working is dM1A1.

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5.

Question Number	Scheme		Marks
5.	$x = 4\cos\left(t+\frac{\pi}{6}\right), \quad y = 2\sin t$		
	Main Scheme		
(a)	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t+\frac{\pi}{6}\right) \rightarrow \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	So, $\{x+y\} = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]

(a)	**Alternative Method 1**	
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$ $\cos\left(t+\frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$	
	So, $x = 2\sqrt{3}\cos t - y$ Forms an equation in x, y and t.	dM1
	$x + y = 2\sqrt{3}\cos t$ * Correct proof	A1 *
		[3]
(b)	**Main Scheme**	
	$\left(\frac{x+y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$ Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \frac{(x+y)^2}{12} + \frac{y^2}{4} = 1$	
	$\Rightarrow (x+y)^2 + 3y^2 = 12$ $(x+y)^2 + 3y^2 = 12$ $\{a = 3, b = 12\}$	A1
		[2]
(b)	**Alternative Method 1**	
	$(x+y)^2 = 12\cos^2 t = 12(1-\sin^2 t) = 12 - 12\sin^2 t$	
	So, $(x+y)^2 = 12 - 3y^2$ Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x+y)^2 + 3y^2 = 12$ $(x+y)^2 + 3y^2 = 12$	A1
		[2]
(b)	**Alternative Method 2**	
	$(x+y)^2 = 12\cos^2 t$	
	As $12\cos^2 t + 12\sin^2 t = 12$	
	then $(x+y)^2 + 3y^2 = 12$	M1, A1
		[2]
		5

		Question 5 Notes
5. (a)	M1	$\cos\left(t+\frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t+\frac{\pi}{6}\right) \to \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$
	Note	**If a candidate states** $\cos(A+B) = \cos A\cos B \pm \sin A\sin B$, but there is an error ***in its application*** then give M1.
		Awarding the dM1 mark which is dependent on the first method mark
Main	dM1	Adds their expanded x (which is in terms of t) to $2\sin t$
	Note	Writing $x + y = ...$ is not needed in the **Main Scheme** method.
Alt 1	dM1	Forms an equation in x, y and t.

	A1*	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x + y\} = 4\cos\left(t + \frac{\pi}{6}\right) + 2\sin t$, by itself is M0M0A0.
(b)	M1 A1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's. leading $(x + y)^2 + 3y^2 = 12$
	SC	Award **Special Case B1B0** for a candidate who writes down **either** • $(x + y)^2 + 3y^2 = 12$ from no working • $a = 3,\ b = 12$, but **<u>does not provide a correct proof</u>**.
	Note **Note**	Alternative method 2 is fine for M1 A1 Writing $(x + y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b \Rightarrow a = 3,\ b = 12$ is SC: B1B0
	Note	Writing $(x + y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b$ • states $a = 3,\ b = 12$ • and refers to either $\cos^2 t + \sin^2 t = 1$ or $12\cos^2 t + 12\sin^2 t = 12$ • and there is no incorrect working would get M1A1

6.

Question Number	Scheme		Marks
7. (a)	$x = 3\tan\theta,\quad y = 4\cos^2\theta$ or $y = 2 + 2\cos 2\theta,\quad 0 \leqslant \theta < \frac{\pi}{2}.$ $\frac{dx}{d\theta} = 3\sec^2\theta,\ \frac{dy}{d\theta} = -8\cos\theta\sin\theta$ **or** $\frac{dy}{d\theta} = -4\sin 2\theta$		
	$\frac{dy}{dx} = \frac{-8\cos\theta\sin\theta}{3\sec^2\theta}\ \left\{= -\frac{8}{3}\cos^3\theta\sin\theta = -\frac{4}{3}\sin 2\theta\cos^2\theta\right\}$	their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ Correct $\frac{dy}{dx}$	M1 A1 oe
	At $P(3, 2)$, $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{8}{3}\cos^3\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)\ \left\{= -\frac{2}{3}\right\}$ So, $m(\mathbf{N}) = \frac{3}{2}$	Some evidence of substituting $\theta = \frac{\pi}{4}$ into their $\frac{dy}{dx}$ applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1 M1
	Either **N:** $y - 2 = \text{"}\frac{3}{2}\text{"}(x - 3)$ **or** $2 = \left(\text{"}\frac{3}{2}\text{"}\right)(3) + c$	**see notes**	M1

	{At Q, $y = 0$, so, $-2 = \frac{3}{2}(x-3)$ } **giving** $\underline{x = \frac{5}{3}}$ $x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67	A1 **cso**
		[6]
(b)	$\left\{\int y^2 \mathrm{d}x = \int y^2 \frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta\right\} = \left\{\int\right\}(4\cos^2\theta)^2 3\sec^2\theta\ \{\mathrm{d}\theta\}$ **see notes**	M1
	So, $\pi\int y^2 \mathrm{d}x = \pi\int(4\cos^2\theta)^2 3\sec^2\theta\ \{\mathrm{d}\theta\}$ **see notes**	A1
	$\int y^2 \mathrm{d}x = \int 48\cos^2\theta\,\mathrm{d}\theta$ $\int 48\cos^2\theta\{\mathrm{d}\theta\}$	A1
	$= \{48\}\int\left(\frac{1+\cos 2\theta}{2}\right)\mathrm{d}\theta\ \ \left\{= \int(24 + 24\cos 2\theta)\mathrm{d}\theta\right\}$ Applies $\cos 2\theta = 2\cos^2\theta - 1$	M1
	$= \{48\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)\ \ \{= 24\theta + 12\sin 2\theta\}$ **Dependent on the first method mark**. For $\pm\alpha\theta \pm \beta\sin 2\theta$	dM1
	$\cos^2\theta \to \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$	A1
	$\int_0^{\frac{\pi}{4}} y^2 \mathrm{d}x \left\{= 48\left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{4}}\right\} = \{48\}\left(\left(\frac{\pi}{8} + \frac{1}{4}\right) - (0+0)\right)\ \{= 6\pi + 12\}$ **Dependent on the third method mark.**	dM1
	{So $V = \pi\int_0^{\frac{\pi}{4}} y^2 \mathrm{d}x = 6\pi^2 + 12\pi$ }	
	$V_{cone} = \frac{1}{3}\pi(2)^2\left(3 - \frac{5}{3}\right)\ \left\{= \frac{16\pi}{9}\right\}$ $V_{cone} = \frac{1}{3}\pi(2)^2(3 - \text{their } (a))$	M1
	$\left\{\text{Vol}(S) = 6\pi^2 + 12\pi - \frac{16\pi}{9}\right\} \Rightarrow \text{Vol}(S) = \underline{\frac{92}{9}\pi + 6\pi^2}$ $\frac{92}{9}\pi + 6\pi^2$	A1
	$\left\{p = \frac{92}{9},\ q = 6\right\}$	**[9]**
		15

		Question 7 Notes
7. (a)	**1[st] M1**	Applies their $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ divided by their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ **or** applies $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ multiplied by their $\frac{\mathrm{d}\theta}{\mathrm{d}x}$
	SC	Award **Special Case 1[st] M1** if **both** $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ **and** $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ are both correct.
	1[st] A1	Correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ i.e. $\frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ or $-\frac{8}{3}\cos^3\theta\sin\theta$ or $-\frac{4}{3}\sin 2\theta\cos^2\theta$ or any equivalent form.
	2[nd] M1	***Some evidence*** of substituting $\theta = \frac{\pi}{4}$ or $\theta = 45^\circ$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Note	For 3[rd] M1 and 4[th] M1, $m(\mathbf{T})$ must be found by using $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	3[rd] M1	applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$. Numerical value for $m(\mathbf{N})$ is required here.
	4[th] M1	• Applies $y - 2 = (\text{their } m_N)(x - 3)$, where m(**N**) is a numerical value, • or ***finds c*** by solving $2 = (\text{their } m_N)3 + c$, where m(**N**) is a numerical value, and $m_N = -\frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = -$ their m(**T**).
	Note	This mark can be implied by subsequent working.

	2nd A1	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67 **from a correct solution only.**
(b)	1st M1	Applying $\int y^2 \mathrm{d}x$ as $y^2 \frac{\mathrm{d}x}{\mathrm{d}\theta}$ with their $\frac{\mathrm{d}x}{\mathrm{d}\theta}$. Ignore π or $\frac{1}{3}\pi$ outside integral.
	Note	You can ignore the omission of an integral sign and/or $\mathrm{d}\theta$ for the 1st M1.
	Note	Allow 1st M1 for $\int (\cos^2\theta)^2 \times$ "their $3\sec^2\theta$" $\mathrm{d}\theta$ or $\int 4(\cos^2\theta)^2 \times$ "their $3\sec^2\theta$ " $\mathrm{d}\theta$
	1st A1	Correct expression $\left\{\pi \int y^2 \mathrm{d}x\right\} = \pi \int (4\cos^2\theta)^2\, 3\sec^2\theta\, \{\mathrm{d}\theta\}$ (Allow the omission of $\mathrm{d}\theta$)
	Note	**IMPORTANT:** The π can be recovered later, **but as a correct statement only.**
	2nd A1	$\left\{\int y^2 \mathrm{d}x\right\} = \int 48\cos^2\theta \{\mathrm{d}\theta\}$. (Ignore $\mathrm{d}\theta$). **Note:** 48 can be written as 24(2) for example.
	2nd M1	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral. (Seen or **implied.**)
	3rd dM1*	**which is dependent on the 1st M1 mark.** Integrating $\cos^2\theta$ to give $\pm\alpha\theta \pm \beta\sin 2\theta$, $\alpha \neq 0$, $\beta \neq 0$, un-simplified or simplified.
	3rd A1	**which is dependent on the 3rd M1 mark and the 1st M1 mark.** Integrating $\cos^2\theta$ to give $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$, un-simplified or simplified. This can be implied by $k\cos^2\theta$ giving $\frac{k}{2}\theta + \frac{k}{4}\sin 2\theta$, un-simplified or simplified.
	4th dM1	**which is dependent on the 3rd M1 mark and the 1st M1 mark.** Some evidence of applying limits of $\frac{\pi}{4}$ and 0 (0 can be implied) to an integrated function in θ
	5th M1	Applies $V_{\text{cone}} = \frac{1}{3}\pi(2)^2(3 - \text{their part}(a)\text{ answer})$.
	Note	Also allow the 5th M1 for $V_{\text{cone}} = \pi \int_{\text{their}\,\frac{5}{3}}^{3} \left(\frac{3}{2}x - \frac{5}{2}\right)^2 \{dx\}$, which includes the correct limits.
	4th A1	$\frac{92}{9}\pi + 6\pi^2$ or $10\frac{2}{9}\pi + 6\pi^2$
	Note	A decimal answer of 91.33168464... (without a correct **exact** answer) is A0.
	Note	The π in the volume formula is only needed for the 1st A1 mark and the final accuracy mark.
7.		<u>**Working with a Cartesian Equation**</u> A cartesian equation for C is $y = \frac{36}{x^2 + 9}$
(a)	1st M1	$\frac{\mathrm{d}y}{\mathrm{d}x} = \pm\lambda x(\pm\alpha x^2 \pm \beta)^{-2}$ **or** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\pm\lambda x}{(\pm\alpha x^2 \pm \beta)^2}$
	1st A1	$\frac{\mathrm{d}y}{\mathrm{d}x} = -36(x^2+9)^{-2}(2x)$ **or** $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-72x}{(x^2+9)^2}$ un-simplified or simplified.
	2nd dM1	**Dependent on the 1st M1 mark if a candidate uses this method** For substituting $x = 3$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ i.e. at $P(3, 2)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-72(3)}{(3^2+9)^2} \left\{= -\frac{2}{3}\right\}$ From this point onwards the original scheme can be applied.
(b)	1st M1	For $\int \left(\frac{\pm\lambda}{\pm\alpha x^2 \pm \beta}\right)^2 \{\mathrm{d}x\}$ (π not required for this mark)
	A1	For $\pi \int \left(\frac{36}{x^2+9}\right)^2 \{\mathrm{d}x\}$ (π required for this mark) To integrate, a substitution of $x = 3\tan\theta$ is required which will lead to $\int 48\cos^2\theta\, \mathrm{d}\theta$ and so from this point onwards the original scheme can be applied.

(a)		Another cartesian equation for C is $x^2 = \frac{36}{y} - 9$
	1st M1	$\pm\alpha x = \pm\frac{\beta}{y^2}\frac{dy}{dx}$ or $\pm\alpha x\frac{dx}{dy} = \pm\frac{\beta}{y^2}$
	1st A1	$2x = -\frac{36}{y^2}\frac{dy}{dx}$ or $2x\frac{dx}{dy} = -\frac{36}{y^2}$
	2nd dM1	**Dependent on the 1st M1 mark if a candidate uses this method** For substituting $x = 3$ to find $\frac{dy}{dx}$ i.e. at $P(3, 2)$, $2(3) = -\frac{36}{4}\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \ldots$ From this point onwards the original scheme can be applied.

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7.

Question Number	Scheme		Marks
4.	$x = 2\sin t, \quad y = 1 - \cos 2t \quad \{= 2\sin^2 t\}, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$		
(a)	$\frac{dx}{dt} = 2\cos t, \quad \frac{dy}{dt} = 2\sin 2t$ or $\frac{dy}{dt} = 4\sin t\cos t$	At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct.	B1
		Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct.	B1
	So, $\frac{dy}{dx} = \frac{2\sin 2t}{2\cos t} \left\{= \frac{4\cos t\sin t}{2\cos t} = 2\sin t\right\}$	Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ and substitutes $t = \frac{\pi}{6}$ into their $\frac{dy}{dx}$.	M1;
	At $t = \frac{\pi}{6}$, $\frac{dy}{dx} = \frac{2\sin\left(\frac{2\pi}{6}\right)}{2\cos\left(\frac{\pi}{6}\right)}$; $= 1$	Correct value for $\frac{dy}{dx}$ of 1	A1 **cao cso**
			[4]
(b)	$y = 1 - \cos 2t = 1 - (1 - 2\sin^2 t)$ $= 2\sin^2 t$		M1
	So, $y = 2\left(\frac{x}{2}\right)^2$ or $y = \frac{x^2}{2}$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	$y = \frac{x^2}{2}$ or equivalent.	A1 **cso isw**
	Either $k = 2$ or $-2 \leqslant x \leqslant 2$		B1
			[3]
(c)	Range: $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$	See notes	B1 B1
			[2] **9**

Notes for Question 4	
(a)	**B1:** At least one of $\frac{dx}{dt}$ or $\frac{dy}{dt}$ correct. **Note:** that this mark can be implied from their working. **B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are correct. **Note:** that this mark can be implied from their working. **M1:** Applies their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ **and** attempts to substitute $t = \frac{\pi}{6}$ into their expression for $\frac{dy}{dx}$. This mark may be implied by their final answer. Ie. $\frac{dy}{dx} = \frac{\sin 2t}{2\cos t}$ followed by an answer of $\frac{1}{2}$ would be M1 (implied). **A1:** For an answer of 1 ***by correct solution only.*** **Note: Don't just look at the answer! A number of candidates are finding $\frac{dy}{dx} = 1$ from incorrect methods.** **Note: Applying** $\frac{dx}{dt}$ divided by their $\frac{dy}{dt}$ is M0, even if they state $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$. **Special Case: Award SC: B0B0M1A1** for $\frac{dx}{dt} = -2\cos t$, $\frac{dy}{dt} = -2\sin 2t$ leading to $\frac{dy}{dx} = \frac{-2\sin 2t}{-2\cos t}$ which after substitution of $t = \frac{\pi}{6}$, yields $\frac{dy}{dx} = 1$ **Note:** It is possible for you to mark part(a), part (b) and part (c) together. Ignore labelling!

Notes for Question 4 Continued	
4. (b)	**M1:** Uses the **correct** double angle formula $\cos 2t = 1 - 2\sin^2 t$ or $\cos 2t = 2\cos^2 t - 1$ or $\cos 2t = \cos^2 t - \sin^2 t$ in an attempt to get y in terms of $\sin^2 t$ or get y in terms of $\cos^2 t$ or get y in terms of $\sin^2 t$ and $\cos^2 t$. Writing down $y = 2\sin^2 t$ is fine for M1. **A1:** Achieves $y = \frac{x^2}{2}$ or un-simplified equivalents **in the form *y* = f(*x*)**. For example: $y = \frac{2x^2}{4}$ or $y = 2\left(\frac{x}{2}\right)^2$ or $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$ or $y = 1 - \frac{4 - x^2}{4} + \frac{x^2}{4}$ and you can ignore subsequent working if a candidate states a correct version of the Cartesian equation. **IMPORTANT: Please check working as this result can be fluked from an incorrect method.** **Award A0** if there is a $+c$ added to their answer. **B1:** Either $k = 2$ or a candidate writes down $-2 \leqslant x \leqslant 2$. Note: $-2 \leqslant k \leqslant 2$ unless k stated as 2 is B0.
(c)	**Note: The values of 0 and/or 2 need to be evaluated in this part** **B1:** Achieves an inclusive upper **or** lower limit, using acceptable notation. Eg: $f(x) \geqslant 0$ or $f(x) \leqslant 2$ **B1:** $0 \leqslant f(x) \leqslant 2$ or $0 \leqslant y \leqslant 2$ or $0 \leqslant f \leqslant 2$ **Special Case: SC: B1B0** for either $0 < f(x) < 2$ or $0 < f < 2$ or $0 < y < 2$ or $(0, 2)$ **Special Case: SC: B1B0** for $0 \leqslant x \leqslant 2$.

IMPORTANT: Note that: Therefore candidates can use either y or f in place of $\mathrm{f}(x)$

Examples:	$0 \leqslant x \leqslant 2$ is SC: B1B0	$0 < x < 2$ is B0B0
	$x \geqslant 0$ is B0B0	$x \leqslant 2$ is B0B0
	$\mathrm{f}(x) > 0$ is B0B0	$\mathrm{f}(x) < 2$ is B0B0
	$x > 0$ is B0B0	$x < 2$ is B0B0
	$0 \geqslant \mathrm{f}(x) \geqslant 2$ is B0B0	$0 < \mathrm{f}(x) \leqslant 2$ is B1B0
	$0 \leqslant \mathrm{f}(x) < 2$ is B1B0.	$\mathrm{f}(x) \geqslant 0$ is B1B0
	$\mathrm{f}(x) \leqslant 2$ is B1B0	$\mathrm{f}(x) \geqslant 0$ and $\mathrm{f}(x) \leqslant 2$ is B1B1. Must state AND {or} $\cap$
	$2 \leqslant \mathrm{f}(x) \leqslant 2$ is B0B0	$\mathrm{f}(x) \geqslant 0$ or $\mathrm{f}(x) \leqslant 2$ is B1B0.
	$\lvert\mathrm{f}(x)\rvert \leqslant 2$ is B1B0	$\lvert\mathrm{f}(x)\rvert \geqslant 2$ is B0B0
	$1 \leqslant \mathrm{f}(x) \leqslant 2$ is B1B0	$1 < \mathrm{f}(x) < 2$ is B0B0
	$0 \leqslant \mathrm{f}(x) \leqslant 4$ is B1B0	$0 < \mathrm{f}(x) < 4$ is B0B0
	$0 \leqslant$ Range $\leqslant 2$ is B1B0	Range is in between 0 and 2 is B1B0
	$0 <$ Range < 2 is B0B0.	Range $\geqslant 0$ is B1B0
	Range $\leqslant 2$ is B1B0	Range $\geqslant 0$ and Range $\leqslant 2$ is B1B0.
	$[0, 2]$ is B1B1	$(0, 2)$ is SC B1B0

Aliter 4. (a) Way 2	$\frac{\mathrm{d}x}{\mathrm{d}t} = 2\cos t$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 2\sin 2t$, At $t = \frac{\pi}{6}$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 2\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 2\sin\left(\frac{2\pi}{6}\right) = \sqrt{3}$ Hence $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$	So B1, B1. So implied M1, A1.

Notes for Question 4 Continued

Aliter 4. (a) Way 3	$y = \frac{1}{2}x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = x$	Correct differentiation of their Cartesian equation. Finds $\frac{\mathrm{d}y}{\mathrm{d}x} = x$, using the correct Cartesian equation only.	B1ft B1
	At $t = \frac{\pi}{6}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\sin\left(\frac{\pi}{6}\right)$	Finds the value of "x" when $t = \frac{\pi}{6}$ and substitutes this into their $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$= 1$	Correct value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ of 1	A1
Aliter 4. (b) Way 2	$y = 1 - \cos 2t = 1 - (2\cos^2 t - 1)$ $y = 2 - 2\cos^2 t \Rightarrow \cos^2 t = \frac{2-y}{2} \Rightarrow 1 - \sin^2 t = \frac{2-y}{2}$ $1 - \left(\frac{x}{2}\right)^2 = \frac{2-y}{2}$ $y = 2 - 2\left(1 - \left(\frac{x}{2}\right)^2\right)$	M1 (Must be in the form $y = \mathrm{f}(x)$). A1	
Aliter 4. (b) Way 3	$x = 2\sin t \Rightarrow t = \sin^{-1}\left(\frac{x}{2}\right)$ So, $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	Rearranges to make t the subject and substitutes the result into y. $y = 1 - \cos\left(2\sin^{-1}\left(\frac{x}{2}\right)\right)$	M1 A1 **oe**

Aliter 4. (b) Way 5	$\frac{dy}{dx} = 2\sin t = x \Rightarrow y = \frac{1}{2}x^2 + c$ Eg: when eg: $t = 0$ (nb: $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$), $x = 0,\ y = 1 - 1 = 0 \Rightarrow \quad c = 0 \quad \Rightarrow y = \frac{1}{2}x^2$ $\frac{dy}{dx} = x \Rightarrow y = \frac{1}{2}x^2 + c$ Full method of finding $y = \frac{1}{2}x^2$ using a value of t: $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$ **Note:** $\frac{dy}{dx} = 2\sin t = x \Rightarrow y = \frac{1}{2}x^2$, with no attempt to find c is M1A0.	M1 A1

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8.

Question Number	Scheme		Marks
5.	***Working parametrically:*** $x = 1 - \frac{1}{2}t,\quad y = 2^t - 1$ or $y = e^{t\ln 2} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$	Applies $x = 0$ to obtain a value for t.	M1
	When $t = 2$, $y = 2^2 - 1 = 3$	Correct value for y.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^t - 1 \Rightarrow t = 0$	Applies $y = 0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t\ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t\ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$.	M1
	At A, $t =$ "2", so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $t =$ "2" and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	See notes.	M1 A1 oe
			[5]
(d)	$\text{Area}(R) = \int (2^t - 1).\left(-\frac{1}{2}\right)dt$	Complete substitution for both y and dx	M1
	$x = -1 \rightarrow t = 4$ and $x = 1 \rightarrow t = 0$		B1
	$= \left\{-\frac{1}{2}\right\}\left(\frac{2^t}{\ln 2} - t\right)$	Either $2^t \rightarrow \frac{2^t}{\ln 2}$ or $(2^t - 1) \rightarrow \frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $(2^t - 1) \rightarrow \pm\alpha(\ln 2)(2^t) - t$	M1*
		$(2^t - 1) \rightarrow \frac{2^t}{\ln 2} - t$	A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2} - t\right]_4^0\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - 4\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits in t and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6] **15**

5. (a)	**M1:** Applies $x = 0$ and obtains a value of t. **A1:** For $y = 2^2 - 1 = 3$ or $y = 4 - 1 = 3$ ***Alternative Solution 1:*** **M1:** For substituting $t = 2$ into either x or y. **A1:** $x = 1 - \frac{1}{2}(2) = 0$ **and** $y = 2^2 - 1 = 3$ ***Alternative Solution 2:*** **M1:** Applies $y = 3$ and obtains a value of t. **A1:** For $x = 1 - \frac{1}{2}(2) = 0$ or $x = 1 - 1 = 0$. ***Alternative Solution 3:*** **M1:** Applies $y = 3$ or $x = 0$ and obtains a value of t. **A1:** Shows that $t = 2$ for both $y = 3$ and $x = 0$.
(b)	**M1:** Applies $y = 0$ and obtains a value of t. Working must be seen in part (b). **A1:** For finding $x = 1$. **Note:** Award M1A1 for $x = 1$.
(c)	**B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. This mark can be implied by later working. **M1:** Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** their $\frac{dy}{dt}$ must be a function of t. **M1:** Uses their value of t found in part (a) and applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$. **M1:** $y - 3 = (\text{their normal gradient})x$ **or** $y = (\text{their normal gradient})x + 3$ or equivalent. **Note:** Allow M1 for $y - 3 = (\text{their changed tangent gradient})x$ **Note:** Award M0 for $y - 3 = (\text{their tangent gradient})x$. **A1:** $y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or $y - 3 = \frac{1}{\ln 256}(x - 0)$ or $(8\ln 2)y - 24\ln 2 = x$ or $\frac{y-3}{(x-0)} = \frac{1}{8\ln 2}$. You can apply isw here. **Working in decimals** is ok for the three method marks. B1, A1 require exact values.
(d)	**M1:** Complete substitution for both y and dx. So candidate should write down $\int (2^t - 1).\left(\text{their } \frac{dx}{dt}\right)$ **B1:** Changes limits from $x \to t$. $x = -1 \to t = 4$ and $x = 1 \to t = 0$. Note $t = 4$ and $t = 0$ seen is B1. **M1*:** Integrates 2^t correctly to give $\frac{2^t}{\ln 2}$ ... or integrates $(2^t - 1)$ to give either $\frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $\pm\alpha(\ln 2)(2^t) - t$. **A1:** Correct integration of $(2^t - 1)$ with respect to t to give $\frac{2^t}{\ln 2} - t$. **dM1*:** **Depends upon the previous method mark.** Substitutes their limits in t and subtracts either way round. **A1:** Exact answer of $\frac{15}{2\ln 2} - 2$ or $\frac{15}{\ln 4} - 2$ or $\frac{15 - 4\ln 2}{2\ln 2}$ or $\frac{7.5}{\ln 2} - 2$ or $\frac{15}{2}\log_2 e - 2$ or equivalent.

Question Number	Scheme		Marks
5.	***Alternative: Converting to a Cartesian equation:***		
	$t = 2 - 2x \Rightarrow \quad y = 2^{2-2x} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ y = 2^2 - 1$	Applies $x = 0$ in their Cartesian equation...	M1
	$y = 3$	... to arrive at a correct answer of 3.	A1
			[2]
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^{2-2x} - 1 \Rightarrow 0 = 2 - 2x \Rightarrow x = ...$	Applies $y = 0$ to obtain a value for x. (Must be seen in part (b)).	M1
	$x = 1$	$x = 1$	A1
			[2]
(c)	$\frac{dy}{dx} = -2\left(2^{2-2x}\right)\ln 2$	$\pm\lambda 2^{2-2x},\ \lambda \neq 1$	M1
		$-2\left(2^{2-2x}\right)\ln 2$ or equivalent	A1
		(Record M1A1 as B1M1 on ePEN)	
	At A, $x = 0$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $x = 0$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.	***As in the original scheme.***	M1 A1 oe
			[5]
(d)	$\text{Area}(R) = \int \left(2^{2-2x} - 1\right)dx$	Form the integral of their Cartesian equation of C.	M1
	$= \int_{-1}^{1}\left(2^{2-2x} - 1\right)dx$	For $2^{2-2x} - 1$ with limits of $x = -1$ and $x = 1$. Ie. $\int_{-1}^{1}\left(2^{2-2x} - 1\right)$	B1
	$= \left(\frac{2^{2-2x}}{-2\ln 2} - x\right)$	Either $2^{2-2x} \to \frac{2^{2-2x}}{-2\ln 2}$ or $\left(2^{2-2x} - 1\right) \to \frac{2^{2-2x}}{\pm\alpha(\ln 2)} - x$ or $\left(2^{2-2x} - 1\right) \to \pm\alpha(\ln 2)(2^{2-2x}) - x$	M1*
		$\left(2^{2-2x} - 1\right) \to \frac{2^{2-2x}}{-2\ln 2} - x$	A1
	$\left\{\left[\frac{2^{2-2x}}{-2\ln 2} - x\right]_{-1}^{1}\right\} = \left(\left(\frac{1}{-2\ln 2} - 1\right) - \left(\frac{16}{-2\ln 2} + 1\right)\right)$	**Depends on the previous method mark.** Substitutes limits of -1 and their x_B and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6] **15**
(d)	**Alternative method: In Cartesian and applying $u = 2 - 2x$**		
	$\text{Area}(R) = \int \left(2^u - 1\right)\{dx\}$, where $u = 2 - 2x$	**M0**: Unless a candidate ***writes*** $\int \left(2^{2-2x} - 1\right)\{dx\}$	
	$= \int_4^0 \left(2^u - 1\right)\left(-\tfrac{1}{2}\right)\{du\}$	**Then apply the "working parametrically" mark scheme. Ie. This is now M1 B1 ...**	

5. (d) ctd	**Applying the 2nd M1* mark** **M1*:** Integrates 2^t correctly to give $\frac{2^t}{\ln 2}$ or integrates $\left(2^t - 1\right)$ to give either $\frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $\pm\alpha(\ln 2)(2^t) - t$. **M1* :** Integrates $e^{t\ln 2}$ correctly to give $\frac{e^{t\ln 2}}{\ln 2}$ or integrates $\left(e^{t\ln 2} - 1\right)$ to give either $\frac{e^{t\ln 2}}{\pm\alpha(\ln 2)} - t$ or $\pm\alpha(\ln 2)(e^{t\ln 2}) - t$. **M1*:** Integrates 2^{2-2x} correctly to give $\frac{2^{2-2x}}{-2\ln 2}$ or integrates $\left(2^{2-2x} - 1\right)$ to give either $\frac{2^{2-2x}}{\pm\alpha(\ln 2)} - x$ or $\pm\alpha(\ln 2)(2^{2-2x}) - x$. **M1*:** Integrates 2^{A+Bx} correctly to give $\frac{2^{A+Bx}}{B\ln 2}$ or integrates $\left(2^{A+Bx} - 1\right)$ to give either $\frac{2^{A+Bx}}{\pm\alpha(\ln 2)} - x$ or $\pm\alpha(\ln 2)(2^{A+Bx}) - x$.
	Examples Award M1* for $\left(2^t - 1\right) \to \ln 2(2^t) - t$ Award M1* for $\left(2^t - 1\right) \to \frac{2^t}{\ln 2}$ Award M1* for $2^t \to \frac{2^t}{\ln 2}$ Award M0* for $\left(2^t - 1\right) \to 2(2^t) - t$ Award M0* for $\left(2^t - 1\right) \to 2^{t+1} - t$. Award M0* for $\left(2^{2-2x} - 1\right) \to 2^{2(2-2x)} - x$ Award M0* for $\left(2^t - 1\right) \to \frac{2^{t+1}}{(t+1)} - t$ Award M0* for $\left(2^t - 1\right) \to \ln 2(2^t)$ Award M0* for $\left(2^t - 1\right) \to \ln t(2^t) - t$ **Note:** $\int \left(2^t - 1\right).\left(-\frac{1}{2}\right) \mathrm{d}t = \int \frac{1}{2} - 2^{t-1} \mathrm{d}t = \frac{1}{2}t - \frac{2^{t-1}}{\ln 2}$ is fine for M1*A1

Question Number	Scheme		Marks
5. (d)	***Alternative method: For substitution*** $\mathbf{u = 2^t}$		
	$\text{Area}(R) = \int (2^t - 1).\left(-\frac{1}{2}\right)\text{d}t$	Complete substitution for both y and $\text{d}x$	M1
	where $u = 2^t \Rightarrow \frac{\text{d}u}{\text{d}t} = 2^t \ln 2 \Rightarrow \frac{\text{d}u}{\text{d}t} = u\ln 2$		
	$x = -1 \to t = 4 \to u = 16$ and $x = 1 \to t = 0 \to u = 1$	Both correct limits in t or both correct limits in u.	B1
	So $\text{area}(R) = -\frac{1}{2}\int \frac{u-1}{u\ln 2}\text{d}u$ $= -\frac{1}{2}\int \frac{1}{\ln 2} - \frac{1}{u\ln 2}\text{d}u$	If not awarded above, you can award M1 for this integral	
	$= \left\{-\frac{1}{2}\right\}\left(\frac{u}{\ln 2} - \frac{\ln u}{\ln 2}\right)$	Either $2^t \to \frac{u}{\ln 2}$ or $(2^t - 1) \to \frac{u}{\pm\alpha(\ln 2)} - \frac{\ln u}{\ln 2}$ or $(2^t - 1) \to \pm\alpha(\ln 2)(u) - \frac{\ln u}{\ln 2}$	M1*
		$(2^t - 1) \to \frac{u}{\ln 2} - \frac{\ln u}{\ln 2}$	A1
	$\left\{-\frac{1}{2}\left[\frac{u}{\ln 2} - \frac{\ln u}{\ln 2}\right]_{16}^{1}\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - \frac{\ln 16}{\ln 2}\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits ***in u*** and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - \frac{\ln 16}{2\ln 2}$ or $\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6]

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9.

Question Number	Scheme	Marks
6.	(a) $\frac{\text{d}x}{\text{d}t} = 2\sqrt{3}\cos 2t$	B1
	$\frac{\text{d}y}{\text{d}t} = -8\cos t \sin t$	M1 A1
	$\frac{\text{d}y}{\text{d}x} = \frac{-8\cos t\sin t}{2\sqrt{3}\cos 2t}$	M1
	$= -\frac{4\sin 2t}{2\sqrt{3}\cos 2t}$	
	$\frac{\text{d}y}{\text{d}x} = -\frac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\frac{2}{3}\right)$	A1 **(5)**

	(b) When $t=\frac{\pi}{3}$ $\quad x=\frac{3}{2},\ y=1$ can be implied	B1
	$m=-\frac{2}{3}\sqrt{3}\tan\left(\frac{2\pi}{3}\right)\quad(=2)$	M1
	$y-1=2\left(x-\frac{3}{2}\right)$	M1
	$y=2x-2$	A1 **(4)**
	(c) $x=\sqrt{3}\sin 2t=\sqrt{3}\times 2\sin t\cos t$	M1
	$x^2=12\sin^2 t\cos^2 t=12\left(1-\cos^2 t\right)\cos^2 t$	
	$x^2=12\left(1-\frac{y}{4}\right)\frac{y}{4}$ or equivalent	M1 A1 **(3)**
		[12]
	Alternative to (c)	
	$y=2\cos 2t+2$	M1
	$\sin^2 2t+\cos^2 2t=1$	
	$\frac{x^2}{3}+\frac{(y-2)^2}{4}=1$	M1 A1 **(3)**

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10.

Question Number	Scheme	Marks
5.	$x=4\sin\left(t+\frac{\pi}{6}\right),\quad y=3\cos 2t,\quad 0\leqslant t<2\pi$	
(a)	$\frac{dx}{dt}=4\cos\left(t+\frac{\pi}{6}\right),\quad \frac{dy}{dt}=-6\sin 2t$	B1 B1
	So, $\frac{dy}{dx}=\frac{-6\sin 2t}{4\cos\left(t+\frac{\pi}{6}\right)}$	B1$\sqrt{}$ oe
		[3]
(b)	$\left\{\frac{dy}{dx}=0\Rightarrow\right\}\ -6\sin 2t=0$	M1 oe
	@ $t=0,\quad x=4\sin\left(\frac{\pi}{6}\right)=2,\quad y=3\cos 0=3\quad\rightarrow(2,3)$	M1
	@ $t=\frac{\pi}{2},\quad x=4\sin\left(\frac{2\pi}{3}\right)=\frac{4\sqrt{3}}{2},\quad y=3\cos\pi=-3\rightarrow(2\sqrt{3},-3)$	
	@ $t=\pi,\quad x=4\sin\left(\frac{7\pi}{6}\right)=-2,\quad y=3\cos 2\pi=3\rightarrow(-2,3)$	
	@ $t=\frac{3\pi}{2},x=4\sin\left(\frac{5\pi}{3}\right)=\frac{4(-\sqrt{3})}{2},\quad y=3\cos 3\pi=-3\rightarrow(-2\sqrt{3},-3)$	A1A1A1
		[5] **8**

(a)	**B1:** Either one of $\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right)$ or $\frac{dy}{dt} = -6\sin 2t$. They do not have to be simplified. **B1:** Both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ correct. They do not have to be simplified. Any or both of the first two marks can be implied. Don't worry too much about their notation for the first two B1 marks. **B1:** Their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or their $\frac{dy}{dt} \times \frac{1}{\text{their}\left(\frac{dx}{dt}\right)}$. **Note:** This is a follow through mark. *Alternative differentiation in part (a)* $x = 2\sqrt{3}\sin t + 2\cos t \Rightarrow \frac{dx}{dt} = 2\sqrt{3}\cos t - 2\sin t$ $y = 3(2\cos^2 t - 1) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$ or $y = 3\cos^2 t - 3\sin^2 t \Rightarrow \frac{dy}{dt} = -6\cos t\sin t - 6\sin t\cos t$ or $y = 3(1 - 2\sin^2 t) \Rightarrow \frac{dy}{dt} = 3(-4\cos t\sin t)$
5. (b)	**M1:** Candidate sets their numerator from part (a) or their $\frac{dy}{dt}$ equal to 0. Note that their numerator must be a trig function. Ignore $\frac{dx}{dt}$ equal to 0 at this stage. **M1:** Candidate substitutes a found value of t, to attempt to find either one of x or y. The first two method marks can be implied by ONE correct set of coordinates for (x, y) or (y, x) interchanged. A correct point coming from NO WORKING can be awarded M1M1. **A1:** At least TWO sets of coordinates. **A1:** At least THREE sets of coordinates. **A1:** ONLY FOUR correct sets of coordinates. If there are more than 4 sets of coordinates then award A0. **Note:** Candidate can use the diagram's symmetry to write down some of their coordinates. **Note:** When $x = 4\sin\left(\frac{\pi}{6}\right) = 2$, $y = 3\cos 0 = 3$ is acceptable for a pair of coordinates. Also it is fine for candidates to display their coordinates on a table of values. **Note:** The coordinates must be exact for the accuracy marks. Ie (3.46..., -3) or (-3.46..., -3) is A0. **Note:** $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0$ ONLY is fine for the first M1, and potentially the following M1A1A0A0. **Note:** $\frac{dy}{dx} = 0 \Rightarrow \cos t = 0$ ONLY is fine for the first M1 and potentially the following M1A1A0A0. **Note:** $\frac{dy}{dx} = 0 \Rightarrow \sin t = 0$ & $\cos t = 0$ has the potential to achieve all five marks. **Note:** It is possible for a candidate to gain full marks in part (b) if they make sign errors in part (a).
	(b) An alternative method for finding the coordinates of the two maximum points. Some candidates may use $y = 3\cos 2t$ to write down that the y-coordinate of a maximum point is 3. They will then deduce that $t = 0$ or π and proceed to find the x-coordinate of their maximum point. These candidates will receive no credit until they attempt to find one of the x-coordinates for the maximum point. **M1M1:** Candidate states $y = 3$ and attempts to substitute $t = 0$ or π into $x = 4\sin\left(t + \frac{\pi}{6}\right)$. M1M1 can be implied by candidate stating either $(2, 3)$ or $(2, -3)$. **Note:** these marks can only be awarded together for a candidate using this method. **A1:** For both $(2, 3)$ or $(-2, 3)$. **A0A0**: Candidate cannot achieve the final two marks by using this method. They can, however, achieve these marks by subsequently solving their numerator equal to 0.

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11.

Question Number	Scheme	Marks
7.	(a) $\tan\theta = \sqrt{3}$ *or* $\sin\theta = \frac{\sqrt{3}}{2}$	M1
	$\theta = \frac{\pi}{3}$ awrt 1.05	A1 **(2)**
	(b) $\frac{dx}{d\theta} = \sec^2\theta,\ \frac{dy}{d\theta} = \cos\theta$	
	$\frac{dy}{dx} = \frac{\cos\theta}{\sec^2\theta}\ \left(=\cos^3\theta\right)$	M1 A1
	At P, $m = \cos^3\left(\frac{\pi}{3}\right) = \frac{1}{8}$ Can be implied	A1
	Using $mm' = -1$, $m' = -8$	M1
	For normal $y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	M1
	At Q, $y = 0$ $-\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	
	leading to $x = \frac{17}{16}\sqrt{3}$ $\left(k = \frac{17}{16}\right)$ 1.0625	A1 **(6)**
	(c) $\int y^2\,dx = \int y^2 \frac{dx}{d\theta}\,d\theta = \int \sin^2\theta\sec^2\theta\,d\theta$	M1 A1
	$= \int \tan^2\theta\,d\theta$	A1
	$= \int\left(\sec^2\theta - 1\right)d\theta$	M1
	$= \tan\theta - \theta \quad (+C)$	A1
	$V = \pi\int_0^{\frac{\pi}{3}} y^2\,dx = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \pi\left[\left(\sqrt{3} - \frac{\pi}{3}\right) - (0 - 0)\right]$	M1
	$= \sqrt{3}\pi - \frac{1}{3}\pi^2 \qquad \left(p = 1, q = -\frac{1}{3}\right)$	A1 **(7)**
		[15]

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12.

Question Number	Scheme	Marks
6.		
(a)	$\frac{dx}{dt}=\frac{1}{t},\quad \frac{dy}{dt}=2t$	
	$\frac{dy}{dx}=2t^2$	M1 A1
	Using $mm'=-1$, at $t=3$	
	$m'=-\frac{1}{18}$	M1 A1
	$y-7=-\frac{1}{18}(x-\ln 3)$	M1 A1 (6)
(b)	$x=\ln t \Rightarrow t=e^x$	B1
	$y=e^{2x}-2$	M1 A1 (3)
(c)	$V=\pi\int(e^{2x}-2)^2\,dx$	M1
	$\int(e^{2x}-2)^2\,dx=\int(e^{4x}-4e^{2x}+4)\,dx$	M1
	$=\frac{e^{4x}}{4}-\frac{4e^{2x}}{2}+4x$	M1 A1
	$\pi\left[\frac{e^{4x}}{4}-\frac{4e^{2x}}{2}+4x\right]_{\ln 2}^{\ln 4}=\pi[(64-32+4\ln 4)-(4-8+4\ln 2)]$	M1
	$=\pi(36+4\ln 2)$	A1
		(6) **[15]**
	Alternative to (c) *using parameters*	
	$V=\pi\int(t^2-2)^2\frac{dx}{dt}\,dt$	M1
	$\int\left((t^2-2)^2\times\frac{1}{t}\right)dt=\int\left(t^3-4t+\frac{4}{t}\right)dt$	M1
	$=\frac{t^4}{4}-2t^2+4\ln t$	M1 A1
	The limits are $t=2$ and $t=4$	
	$\pi\left[\frac{t^4}{4}-2t^2+4\ln t\right]_2^4=\pi[(64-32+4\ln 4)-(4-8+4\ln 2)]$	M1
	$=\pi(36+4\ln 2)$	A1
		(6)

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13.

Question Number	Scheme	Marks
4.	(a) $\dfrac{dx}{dt} = 2\sin t\cos t,\ \dfrac{dy}{dt} = 2\sec^2 t$	B1 B1
	$\dfrac{dy}{dx} = \dfrac{\sec^2 t}{\sin t\cos t}\left(= \dfrac{1}{\sin t\cos^3 t}\right)$ or equivalent	M1 A1 **(4)**
	(b) At $t = \dfrac{\pi}{3},\ x = \dfrac{3}{4},\ y = 2\sqrt{3}$	B1
	$\dfrac{dy}{dx} = \dfrac{\sec^2 \frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \dfrac{16}{\sqrt{3}}$	M1 A1
	$y - 2\sqrt{3} = \dfrac{16}{\sqrt{3}}\left(x - \dfrac{3}{4}\right)$	M1
	$y = 0 \Rightarrow x = \dfrac{3}{8}$	M1 A1 **(6)**
		[10]

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14.

Question Number	Scheme		Marks	
Q7	(a) $y=0 \Rightarrow t(9-t^2)=t(3-t)(3+t)=0$			
	$t=0, 3, -3$	Any one correct value	B1	
	At $t=0$, $x=5(0)^2-4=-4$	Method for finding one value of x	M1	
	At $t=3$, $x=5(3)^2-4=41$			
	$\left(\text{At } t=-3,\ x=5(-3)^2-4=41\right)$			
	At A, $x=-4$; at B, $x=41$	Both	A1	(3)
	(b) $\dfrac{dx}{dt}=10t$	Seen or implied	B1	
	$\int y\,dx=\int y\dfrac{dx}{dt}\,dt=\int t(9-t^2)10t\,dt$		M1 A1	
	$=\int(90t^2-10t^4)\,dt$			
	$=\dfrac{90t^3}{3}-\dfrac{10t^5}{5}\ (+C)\quad (=30t^3-2t^5\ (+C))$		A1	
	$\left[\dfrac{90t^3}{3}-\dfrac{10t^5}{5}\right]_0^3=30\times3^3-2\times3^5\quad(=324)$		M1	
	$A=2\int y\,dx=648\quad(\text{units}^2)$		A1	(6)
				[9]